A Ballistic Similitude Design Criterion for Artillery Projectiles

Albert E. Hodapp Jr.* and Robert A. LaFarge†
Sandia National Laboratories
Albuquerque, New Mexico

Introduction

THIS Note describes the analytically derived and experimentally verified ballistic similitude design criterion (BSDC) that was used to guide development of the M753 8-in. artillery-fired atomic projectile (AFAP) as a ballistically similar counterpart of the M650 rocket-assisted (RA) 8-in. conventional high-explosive (HE) projectile. Large differences in internal configuration between an AFAP and a conventional HE projectile of identical shape make it impossible to adjust all mass properties to match. Therefore, a BSDC was required to define the mass properties that must be matched. This BSDC, which is applicable to projectiles of identical external shape like the M753 and M650 (Fig. 1), also can be used to ballistically match conventional projectiles that carry different payloads.

An AFAP meets Army ballistic similitude requirements with a conventional base projectile if its mean point of impact falls within the "precision error" region about the conventional projectile mean impact point (Fig. 2) when the projectiles are fired with identical propelling charges at conditions that can differ only by small differences in quadrant elevation (QE) and azimuth angles. A U.S. Army ballistic similitude definition establishes the corrections that are allowed to determine these differences. The corrections for the M753 were determined from data obtained during a ballistic similitude verification test conducted at Yuma Proving Ground (YPG), Arizona, during fall 1979. The small range probable error (RPE) and deflection probable error (DPE) components of the M650 precision error, listed in Fig. 2 for maximum and minimum values of range and deflection, indicate how restrictive the ballistic similitude requirement is.

Similitude Requirements

The range of a spin-stabilized projectile is determined by the muzzle velocity U, QE angle, and effective ballistic coefficient. Ballistic coefficient is reduced, hence range is reduced, by the increased drag that results from the projectile's angular motion. The angular motion detected by an Earth-fixed observer (Fig. 3) is composed of damped transient angular motion components (the nutation K_1, and precession K_2 vectors with circular frequencies ω_1 and ω_2) and steady components (the constant-magnitude body-fixed trim vector K_3 with circular frequency p, and the quasisteady fixed yaw of repose K_4). For a given muzzle velocity and QE, deflection depends on projectile weight W and on the β_R component of the yaw of repose K_4 (Fig. 3). Our BSDC for projectiles of identical external shape requires rigid-body characteristics and a match between muzzle velocity, ballistic coefficient, β_R, and mass-asymmetry-induced effects on transient angular motion. Table 1 lists the quantities we have identified to match in order to satisfy these requirements.

Presented as Paper 82-1344 at the AIAA 9th Atmospheric Flight Mechanics Conference, San Diego, Calif., Aug. 9-11, 1982; submitted Aug. 25, 1982; revision received Aug. 23, 1983.

*Distinguished Member of the Technical Staff, Aerodynamics Department. Associate Fellow AIAA.

†Member of the Technical Staff, Aerodynamics Department. Member AIAA.

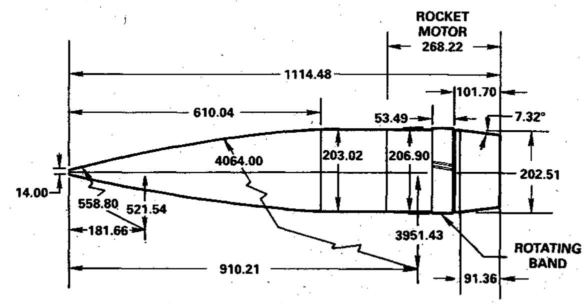

Fig. 1 M650 and M753 external dimensions.

	RANGE, m	RPE, m	DEFLECTION, m	DPE, m
MAXIMUM	30,000	76	2,400	15
MINIMUM	3,000	9	14	2

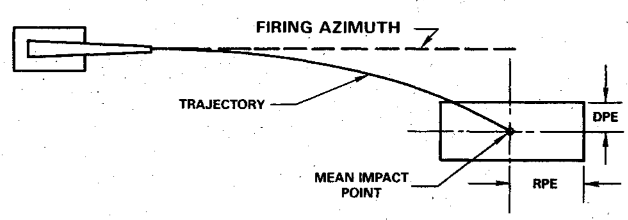

Fig. 2 Trajectory precision error specifications.

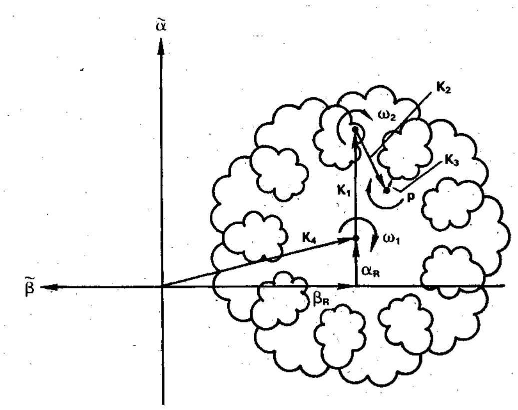

Fig. 3 Projectile angular motion (nonrolling angle of attack $\tilde{\alpha}$ and sideslip $\tilde{\beta}$).

Nonrigid body effects can produce large-range loss and threaten structural integrity. It has been shown[1] that partially restrained bodies within artillery projectiles can cause a serious angular motion instability (K_1 becomes undamped and roll rate p decreases). This instability and the resulting range loss it causes can be avoided by rigidly attaching internal bodies. To ensure structural integrity and stability, the resonances and unstable transient motion that rigidly attached internal bodies can experience must be avoided. This can be accomplished by adjusting the lowest fundamental frequency (critical roll rate p_{cr}) to remain larger than the projectile roll rate p. Therefore, to ensure rigid-body characteristics, internal bodies must be rigidly attached with $p_{cr} > p$.

Small differences in muzzle velocity can have a large effect on range differences between projectiles. For projectiles of identical shape and weight, muzzle velocity can be matched by using the base projectile rotating band with an underlying structure that has response characteristics similar to those of the base projectile. It is important to have similar rotating band characteristics; otherwise, differences in muzzle velocity between the base and matching projectiles can vary with gun wear level.

To match ballistic coefficients the weight W of both projectiles must be matched. Since the projectiles have

Table 1 **BSDC requirements for projectiles of identical external shape**

Characteristics to be matched	Quantities to match	Effects
Rigid body	Rigidly attached internal bodies with $p_{cr} > p$	Range and structural integrity
Muzzle velocity, U	W, Rotating band characteristics	Range
Ballistic coefficient, $W/C_D S$	W	Range
Yaw of repose, β_R	I_X, X_{cg}	Deflection
Mass asymmetry effects	$\lvert\delta\rvert\left\{1+\left(\frac{2I}{I_x}-1\right)\left(\frac{1}{\sqrt{1-1/S_g}}\right)\right\}$	Range

NOTE: Substantial differences in transverse moment of iniertia I can exist between projectiles when the gyroscopic stability factor S_g is sufficiently large.

Table 2 **M753 and M650 mass properties**

Parameters	M650 FCI standard values	M753 test projectiles	M650 projectiles[a]
W, lb (kg)	200.0 (90.72)	200.0 (90.72)	198.9 (90.22)
X_{cg}, in. (m)	29.28 (0.7437)	29.36 (0.7457)	29.22 (0.7422)
I_X, lb-in.^{2} (kg-m^2)	1911 (0.5592)	1903 (0.5569)	1917 (0.5610)
I, lb.-in.^{2} (kg-m^2)	–	15983 (4.6773)	15543 (4.586)
$\lvert\delta\rvert$, deg	–	0.027	0.022

[a] Corrected to a 2.06-lb (0.934-kg) M557 fuze weight.

identical shape, they have the same reference area S. The drag coefficient C_D, which is approximately the same for both projectiles, is a function of shape, surface condition, and magnitude of the projectile angular motion. The BSDC is employed to ensure that differences in C_D are sufficiently small.

If projectiles of the same shape and weight are to have the same yaw-of-response-induced deflection, β_R must be matched. Vaughn and Wilson[2] showed that β_R can be matched by matching the ratio $I_X/(X_{cg}-X_{cp})$. I_X represents the moment of inertia about the projectile's axis of symmetry (X axis). X_{cg} and X_{cp} represent the distances from the nose to the center of gravity and center of pressure, respectively, along the X axis. Because of identical Mach-number-induced variations of X_{cp} that occur for identical shapes, it may not be possible to achieve a sufficiently close match of the above ratio for a full range of flight Mach number conditions when the magnitudes of I_X and X_{cg} are appreciably different from those of the base projectile. Therefore, the most realistic approach for matching deflection is to attempt to match I_X and X_{cg} individually.

As indicated by Vaughn and Wilson,[2] the above restrictions do not include I, the moment of inertia about a transverse reference axis. Six-degree-of-freedom (6-DOF) trajectory simulation results and the firing test results presented in Ref. 3 confirm that deflection differences result from mismatches in I_X and X_{cg} but not from a mismatch in I, while range is insensitive to realistic variations in all of these parameters. This is fortunate because W, I_X, and X_{cg} are AFAP physical characteristics that can be matched closely to those of a conventional base projectile when the requirement for matching I can be relaxed.

A principal axis misalignment angle $\lvert\delta\rvert$ exists (mass asymmetry) when the longitudinal principal axis of a projectile is angularly misaligned with the gun-induced spin axis (ideally the projectile X axis of symmetry). As demonstrated theoretically in Ref. 4 and experimentally in Ref. 5, small differences in $\lvert\delta\rvert$ between otherwise similar projectiles can produce sufficiently large differences in transient angular motion to cause differences in range that can exceed the allowable corrections. Therefore, matching or reducing the effects of $\lvert\delta\rvert$ (Table 1) is an important consideration for ballistic similitude.

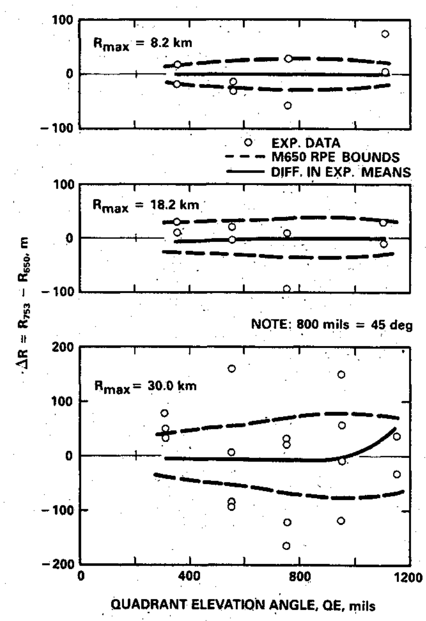

Fig. 4 **M753 ballistic similitude verification test results.**

Similitude Verification

Measured mean mass property values for 55 M753 and 220 M650 projectiles used for the YPG ballistic similitude verification test are given in Table 2, along with the fire control input (FCI) values that define the mean mass properties of the M650. The M753 was developed according to the BSDC (Table 1) to achieve similitude with the M650. It has rigid-body characteristics. The rotating band and support structure are those of the M650. W, I_X, and X_{cg} are all closely matched to those of the FCI standard M650. The small differences in I_X and X_{cg} (Table 2) and the larger difference in I all resulted in correctable differences in mean impact point with the M650. Because M650 $|\delta|$ was negligibly small, M753 $|\delta|$ was reduced to a level that would also yield a negligible effect.

Examples of corrected range differences between the M753 AFAP and HE M650 are given as a function of QE for three separate firing conditions (Fig. 4); i.e., minimum, medium, and maximum range applications. Dashed lines in Fig. 4 are M650 RPE boundaries. Differences in mean impact points (solid lines) were determined by fitting a mathematical model (6-DOF simulation results) to the experimentally determined data points (circular symbols). The results given in Fig. 4 clearly demonstrate that the range differences between the M753 and M650 mean points of impact are less than one M650 RPE. Deflection differences (not shown) are less than one M650 DPE. The results of the ballistic similitude verification test prove that the M753 is ballistically similar to the M650.

Conclusion

The ballistic similitude design criterion (BSDC) presented herein (Table 1) was created to provide a means for developing an artillery-fired atomic projectile (AFAP) as a ballistically similar counterpart to a conventional base projectile. A BSDC is required to identify the critical mass properties for matching, since gross differences in internal configuration make it impossible to adjust all AFAP mass properties. This BSDC was used to guide development of the M753 8-in. AFAP as a ballistically similar counterpart to the M650 8-in. rocket-assisted conventional high-explosive projectile. Results obtained from a ballistic similitude verification test for these projectiles prove that the M753 is ballistically similar to the M650.

Acknowledgment

This work was performed at Sandia National Laboratories, supported by the U.S. Department of Energy under Contract DE-AC04-76DP00789.

References

[1] Murphy, C.H., "Influence of Moving Internal Parts on Angular Motion of Spinning Projectiles," *Journal of Guidance and Control*, Vol. 1, March-April 1978, pp. 117-122.

[2] Vaughn, H.R. and Wilson, G.G., "Effect of Yaw of Repose on Ballistic Match of Similar Projectiles," *AIAA Journal*, Vol. 9, June 1971, pp. 1208-1210.

[3] Rollstin, L.R., "Experimental Determination of the Artillery Shell Mass-Property/Trajectory-Drift Relationship," *Journal of Spacecraft and Rockets*, Vol. 16, March-April 1979, pp. 108-114.

[4] Hodapp, A.E. Jr., "Effect of Mass Asymmetrics on Ballistic Match of Projectiles," *Journal of Spacecraft and Rockets*, Vol. 13, Dec. 1976, pp. 757-760; Errata, Vol. 14, June 1977, p. 384.

[5] Rollstin, L.R. and Hodapp, A.E. Jr., "Experimental Evaluation of Artillery Projectile Impact Errors Induced by Principal Axis Misalignment," *Journal of Spacecraft and Rockets*, Vol. 16, Sept.-Oct. 1979, pp. 316-318.